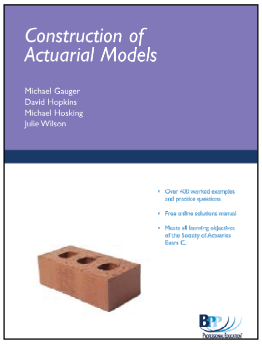

PROFESSIONAL EDUCATION

Construction of Actuarial Models

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Solutions to practice questions – Chapter 4

Solution 4.1

We have:

$$S(x) = \left(\frac{100-x}{100}\right)^{1.02}$$

The PDF is obtained as the negative derivative of the survival function:

$$f(x) = -S'(x) = -1.02\left(\frac{100-x}{100}\right)^{0.02}\left(-\frac{1}{100}\right) = \frac{1.02(100-x)^{0.02}}{100^{1.02}}$$

Solution 4.2

$$h(x) = 2/(75+x)\ ,\ x \geq 0 \ \Rightarrow\ H(x) = \int_0^x \frac{2}{75+y}\,dy = 2\ln\left(\frac{75+x}{75}\right) \ \Rightarrow$$

$$S(x) = \exp(-H(x)) = \exp\left(-2\ln\left(\frac{75+x}{75}\right)\right) = \left(\frac{75}{75+x}\right)^2 \quad \text{for } x > 0$$

Solution 4.3

$$f(x) = -S'(x) = -\left(\frac{75^2}{(75+x)^2}\right)' = \frac{2\times 75^2}{(75+x)^3} \quad \text{for } x > 0$$

Solution 4.4

$$E[X] = \int_0^\infty S(x)\,dx = \int_0^{75} \frac{75^2}{(75+x)^2}\,dx = -\frac{75^2}{75+x}\Bigg|_0^\infty = -0 + 75 = 75$$

Solution 4.5

We have $\mu = E[X] = \int_0^\infty S(x)\,dx$, so it would be appealing to imitate this relationship:

$$\hat{\mu} = \int_0^\infty \hat{S}(x)\,dx$$

Solution 4.6

$$h(1) = \frac{\Pr(X=1)}{\Pr(X \ge 1)} = \frac{0.4}{1.0}\ ,\quad h(2) = \frac{\Pr(X=2)}{\Pr(X \ge 2)} = \frac{0.6}{0.6}\ ,\quad \text{zero otherwise}$$

Solution 4.7

In general we have $H(x) = \sum_{x_i \le x} h(x)$. So we have:

$$H(1.2) = h(1) = 0.4\ ,\ H(2) = h(1) + h(2) = 1.4\ ,\ H(2.5) = h(1) + h(2) = 1.4$$

Solution 4.8

There are 6 of the 8 lives surviving at time 1.2. So we have $\hat{S}(1.2) = 6/8 = 0.75$.

The exact and approximate variances of this estimator are:

$$\text{var}\left(\hat{S}(1.2)\right) = \frac{S(1.2)\left(1 - S(1.2)\right)}{8} \approx \frac{0.75 \times 0.25}{8} = 0.02344$$

Solution 4.9

The empirical distribution has $f_2(1) = f_2(1.8) = 0.5$. Using $b = 1$, we have:

$$k_1^t(x) = \frac{b - |1-x|}{b^2} = 1 - |1-x| \ \text{ for } \ 0 \le x \le 2 \ \Rightarrow \ k_1^t(1.2) = 0.8$$

$$k_{1.8}^t(x) = \frac{b - |1.8-x|}{b^2} = 1 - |1.8-x| \ \text{ for } \ 0.8 \le x \le 2.8 \ \Rightarrow \ k_{1.8}^t(1.2) = 0.4$$

So the kernel smoother approximation of $f(1.2)$ is:

$$f_2^{ks}(1.2) = \sum_{i=1}^{2} \frac{1}{2} k_{x_i}(1.2) = \frac{1}{2} \times 0.8 + \frac{1}{2} \times 0.4 = 0.6$$

Solution 4.10

For this question, we have:

$$F_2^{ks}(1.2) = \int_{-\infty}^{1.2} f_2^{ks}(y)\, dy = \sum_{i=1}^{2} \frac{1}{2} K_{x_i}(1.2)$$

where:

$$K_1^t(1.2) = \underbrace{\int_0^1 k_1^t(x)\,dx}_{1/2 \ \text{(symmetry)}} + \int_1^{1.2} k_1^t(x)\,dx = 0.5 + \int_1^{1.2} 1-|1-x|\,dx$$

$$= 0.5 + \int_1^{1.2} 2-x\,dx = 0.5 + 0.18 = 0.68$$

$$K_{1.8}^t(1.2) = \int_{0.8}^{1.2} k_{1.8}^t(x)\,dx = \int_{0.8}^{1.2} 1-|1.8-x|\,dx$$

$$= \int_{0.8}^{1.2} x-0.8\,dx = 0.4 - 0.32 = 0.08$$

Finally, we have:

$$F_2^{ks}(1.2) = \sum_{i=1}^{2} \frac{1}{2} K_{x_i}(1.2) = \frac{1}{2}(0.68 + 0.08) = 0.38$$