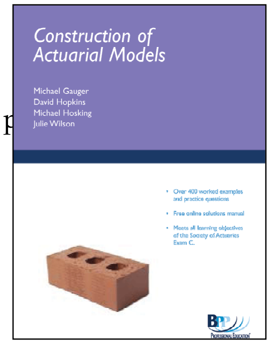

Construction of Actuarial Models

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Solutions to practice questions – Chapter 9

Solution 9.1

We have $\bar{x} = \dfrac{100+120+110}{3} = 110$ and $\mu = 90$. It follows that the insurer has set $Z = 50\%$.

Solution 9.2

(a) If the average variability of the claim amounts for each risk increases, then the direct data is less likely to be typical of the risk, and so less credible, *ie* Z will fall.

(b) If the variability of the mean claim amount for each risk increases, then the collateral data is less representative of the actual risk of interest, and hence the collateral data is less credible, so the direct data must be more credible, *ie* Z will increase.

Solution 9.3

Since 500 is fewer than 1000, it must be the case that the credibility factor is less than 50%, though we cannot give a precise figure without more information. With 1500 policies, the credibility will lie between 50% and 100%, although again we cannot be any more precise. With 3000 policies we will have full credibility.

Solution 9.4

With only 5 claims we would have:

$$\begin{aligned} n &\geq \left(\frac{1}{k\sqrt{n}}\Phi^{-1}\left(\frac{1+p}{2}\right)\right)^2 \\ &= \left(\frac{1}{0.1\sqrt{(5/5)}}\Phi^{-1}\left(\frac{1+0.95}{2}\right)\right)^2 \\ &= \left(10\Phi^{-1}(0.975)\right)^2 \\ &= 100\times(1.96)^2 \\ &= 384.16 \end{aligned}$$

We therefore now require at least 385 years of data for full credibility(!)
It is unlikely that for an auto policy on a single vehicle there would ever be enough data on that policy to make it fully credible, since the incidence rate of claims is so low. A single group policy might reach full credibility.

Another problem in this case is that the estimate 385 is less likely to be reliable; the standard for full credibility would ideally be determined using the true underlying mean μ. Since this is unknown we have estimated it, but since we don't have much data, this estimate will be less reliable in general.

Solution 9.5

We require:

$$n\bar{n} \geq \left(\frac{1}{k}\Phi^{-1}\left(\frac{1+p}{2}\right)\right)^2 = \left(20\Phi^{-1}\left(\frac{1+0.90}{2}\right)\right)^2 = 400 \times (1.645)^2 = 1082.41$$

You may see the figure of 1083 (or 1082) often quoted in credibility texts as the minimum figure needed for full credibility in this situation.

Solution 9.6

If we assume that the number of claims on each policy is $Binomial(10, q)$ then the total number of claims $N \sim Binomial(20000, q)$. The parameter q can be estimated to be $\hat{q} = \frac{1250}{20000} = 0.0625$. It follows that we require:

$$\begin{aligned} nmq &\geq \left(\frac{\sqrt{(1-q)}}{k}\Phi^{-1}\left(\frac{1+p}{2}\right)\right)^2 \\ &= \left(\frac{\sqrt{(1-0.0625)}}{0.1}\Phi^{-1}\left(\frac{1+0.90}{2}\right)\right)^2 \\ &= \left(10 \times \sqrt{0.9375} \times 1.645\right)^2 \\ &= 253.7 \end{aligned}$$

So the standard for full credibility is an expected number of claims of 254.

Solution 9.7

We need:

$$n \geq \left(\frac{\sqrt{s^2}}{k\bar{x}}\Phi^{-1}\left(\frac{1+p}{2}\right)\right)^2 = \left(\frac{200}{0.1 \times 980}\Phi^{-1}\left(\frac{1+0.95}{2}\right)\right)^2 = 16$$

Solution 9.8

The annual aggregate claims follow a compound Poisson distribution with mean $\lambda = \frac{1670}{5} = 334$. We also have $E[X] = 150$ and $\text{var}(X) = 70^2$.

Therefore, they require:

$$n\lambda \geq \left(\frac{\sqrt{E[X^2]}}{kE[X]} \Phi^{-1}\left(\frac{1+p}{2}\right) \right)^2 = \left(\frac{\sqrt{150^2 + 70^2}}{0.05 \times 150} \Phi^{-1}\left(\frac{1+0.95}{2}\right) \right)^2 = 1871.29$$

Solution 9.9

The direct data is only partially credible. The square root rule suggests a credibility factor of:

$$Z = \sqrt{\frac{50}{200}} = 0.5$$

Hence the credibility estimate will be:

$$0.5 \times \frac{12500}{50} + 0.5 \times 200 = \$225$$

Solution 9.10

We want:

$$10 = \left(\frac{1}{0.15\sqrt{8.7}} \Phi^{-1}\left(\frac{1+p}{2}\right) \right)^2$$

Rearranging this equation, we find that:

$$\Phi^{-1}\left(\frac{1+p}{2}\right) = 1.399$$

But from the tables of the normal distribution, we have: $\Phi(1.399) = 0.91909$. So the correct value of p is 0.838.

Solution 9.11

We now need to solve the equation:

$$10 = \left(\frac{1}{k\sqrt{8.7}} \Phi^{-1}(0.98) \right)^2$$

From the tables of the normal distribution, we have: $\Phi^{-1}(0.98) = 2.0537$. Solving this equation for k, we obtain $k = 0.2202$, *ie* about 22%.

Solution 9.12

No. Compare with the previous question. We have found that $k = 0.22$ and $p = 0.96$ gives a combination of values that is just fully credible. If we choose a higher value for p (0.98), we then need to broaden our interval in order to achieve full credibility as before. So for $p = 0.98$ we will need a larger value of k than $k = 0.22$. Here our value of k is smaller ($k = 0.20$). So full credibility will not be achieved here.

Solution 9.13

Here we need:

$$n\bar{n} > \left(\frac{1}{0.01}\Phi^{-1}(0.995)\right)^2 = (100 \times 2.5758)^2 = 66,347.46$$

So the number of claims needed is at least 66,348.

Solution 9.14

The mean of the data sample is 16.2. So our estimate for q is:

$$\hat{q} = \frac{16.2}{100} = 0.162$$

So we now require:

$$n\bar{n} \geq \left(\frac{\sqrt{\left(1-\left(\bar{n}/m\right)\right)}}{k}\Phi^{-1}\left(\frac{1+p}{2}\right)\right)^2$$

where here we have $\bar{n} = 16.2$, $m = 100$.

So we want:

$$n\bar{n} \geq \left(\frac{\sqrt{(1-0.162)}}{0.1}\Phi^{-1}(0.99)\right)^2 = 453.49$$

But since $\bar{n} = 16.2$, this gives:

$$n \geq \frac{453.49}{16.2} = 27.99$$

So we need a sample size of at least 28.

Solution 9.15

The sample estimates are:

$$\bar{n} = 2.8 \qquad s_n^2 = \frac{1}{4}\left(46 - 5(2.8)^2\right) = 1.7$$

$$\bar{x} = \frac{3,940}{14} = 281.429 \qquad s_x^2 = \frac{1}{13}\left(1,256,400 - 14(281.429)^2\right) = 11,351.649$$

So our estimates of the mean and variance of S are:

$$\hat{\mu} = 2.8 \times 281.429 = 788$$

$$\hat{\sigma}^2 = 281.429^2 \times 1.7 + 11,351.649 \times 2.8 = 166,428$$

This gives an estimate for the standard deviation of S of 407.956.

So the number of years' past data needed here would be:

$$n \geq \left[\frac{407.956}{0.1 \times 788}\Phi^{-1}(0.975)\right]^2 = 102.964$$

So at least 103 years of past data would be needed.

Solution 9.16

This is similar to the previous question, but our estimates are now:

$$\hat{\mu} = \frac{430 + \cdots + 1,410}{5} = 788 \qquad \text{(as before)}$$

and: $$\hat{\sigma}_s^2 = \frac{1}{4}\left(4,082,200 - 5(788)^2\right) = 244,370$$

This gives an estimate of the standard deviation of $\hat{\sigma}_s = 494.338$.

Using the same formula as in the previous question, we obtain that the number of past years' data needed is:

$$\left(\frac{494.338}{0.1 \times 788} \times 1.96\right)^2 = 151.18$$

Solution 9.17

Both students need many more years of data than they actually have. In neither case is the data at all credible.

However, Student B needs more past years' data than Student A. This is because his estimate for the variance is greater, which means that he is more uncertain about the level of claims underlying the data. To combat this greater uncertainty, he needs to look at more past data to achieve the same level of credibility.

Solution 9.18

We have, for the negative binomial and Pareto distributions:

$$E(N) = r\beta = 2.4$$

$$\text{var}(N) = r\beta(1+\beta) = 3.84$$

$$E(X) = \frac{\theta}{\alpha - 1} = 300$$

$$\text{var}(X) = \frac{\alpha\theta^2}{(\alpha-1)^2(\alpha-2)} = 112,500$$

We can now calculate the mean and variance of S directly:

$$E(S) = E(X)E(N) = 300 \times 2.4 = 720$$

and:

$$\text{var}(S) = \left[E(X)\right]^2 \text{var}(N) + \text{var}(X)E(N) = 300^2(3.84) + 112,500(2.4) = 615,600$$

So the mean and standard deviation of S are 720 and 784.602 respectively.

So the number of years of past data required is now:

$$\left(\frac{784.602}{0.1 \times 720} \times 1.96\right)^2 = 456.19$$

So 457 years of past data would now be required.

Solution 9.19

Again the amount of past data required is very large. Again the increase is due to the greater assumed variance in Student C's calculations. Of course, Student C appears not to be using the sample data at all, so it would be worth asking where Student C's assumptions are coming from!

Solution 9.20

The mean and variance of the compound distribution are:

$$E(S) = \lambda m$$

and: $$\text{var}(S) = \lambda(m^2 + s^2)$$

Since we have a single value y for the aggregate amount paid out in a year, we can write:

$$0.95 = \Pr\left[0.9E(S) \le y \le 1.1E(S)\right] = \Pr\left[-\frac{0.1E(S)}{\sqrt{\text{var}(S)}} \le N(0,1) \le \frac{0.1E(S)}{\sqrt{\text{var}(S)}}\right]$$

Equating the expression on the right hand side of the inequality to 1.96, we have:

$$\frac{0.1\lambda m}{\sqrt{\lambda(m^2+s^2)}} = 1.96$$

Rearranging this, we obtain:

$$\sqrt{\lambda} = \frac{1.96\sqrt{m^2 + s^2}}{0.1m}$$

Squaring this and simplifying, we get:

$$\lambda = 384.16\ (1 + s^2 / m^2)$$

This is the required expression for λ.

If X is lognormal:

$$m = e^{\mu + ½\sigma^2} = e^{4.8} = 121.51$$

and $s^2 = e^{2\mu + \sigma^2}(e^{\sigma^2} - 1) = e^{9.6}(e^{1.6} - 1) = 58,365.66$

So the minimum value of λ is now:

$$\lambda = 384.16\left(1 + \frac{58,365.66}{121.51^2}\right) = 1,902.77$$

The ratio s/m is known as the **coefficient of variation** of the claim amount distribution.