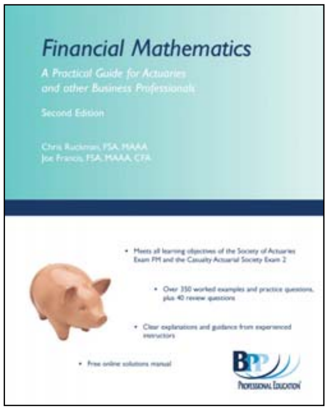

Financial Mathematics
Second Edition

A Practical Guide for Actuaries and other Business Professionals

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Solutions to practice questions – Chapter 1

Solution 1.1

After 6 years, the accumulated value is:

$$300 \times (1 + 6 \times 0.035) = \$363.00$$

Solution 1.2

For Fund P, the accumulated amount after 5 years is:

$$100(1 + 5 \times 0.04) = 120.00$$

For Fund Q, the accumulated amount after 5 years is:

$$118.5(1 + 5i)$$

These must be equal, so:

$$120 = 118.5(1 + 5i)$$
$$\Rightarrow \quad i = 0.00253 \quad \text{or} \quad i = 0.253\%$$

Solution 1.3

The accumulated value after 15 years is:

$$800 \times 1.0525^{15} = \$1,723.54$$

Solution 1.4

For Fund P, the accumulated amount after 5 years is:

$$100(1.04)^5$$

For Fund Q, the accumulated amount after 5 years is:

$$118.5(1+i)^5$$

These must be equal, so:

$$100(1.04)^5 = 118.5(1+i)^5$$
$$\Rightarrow \quad i = 0.00529 \quad \text{or} \quad i = 0.529\%$$

Solution 1.5

The present value is:

$$5,000v^{20} = 5,000(1.05)^{-20} = \$1,884.45$$

Solution 1.6

The money has been invested for $t = 127.5$ years.

Assuming a 5% annual effective interest rate, the initial $10 deposit would have grown to:

$$10(1.05)^{127.5} = \$5,030.78$$

Assuming a 10% annual effective interest rate, the initial $10 deposit would have grown to:

$$10(1.10)^{127.5} = \$1,894,817.37$$

This illustrates the power of compound interest. Doubling the interest rate from 5% to 10% causes the accumulated value to increase much greater than two times due to the fact that the extra interest is compounded over such a long period of time.

Solution 1.7

If Larry waits 18 months to pay off the credit card balance in full, he will owe:

$$10,000(1.15)^{1.5} = \$12,332.38$$

If Larry takes out the home equity loan, pays off the credit card balance today, and then repays the home equity loan in full 18 months from today, he will owe:

$$10,000(1.05)^{1.5} = \$10,759.30$$

This strategy saves Larry $12,332.38 - 10,759.30 = \$1,573.08$.

Solution 1.8

The accumulated value is:

$$5{,}000(1-d)^{-t} = 5{,}000(1-0.05)^{-20} = 5{,}000(0.95)^{-20} = \$13{,}947.55$$

Solution 1.9

The present value is:

$$5{,}000\left(1-td\right) = 5{,}000(1-0.25\times 0.04) = \$4{,}950.00$$

Solution 1.10

We have:

$$15{,}000 = 18{,}375.65(1-d)^3$$

$$\Rightarrow 1-d = \left(\frac{15{,}000}{18{,}375.65}\right)^{1/3} \quad \Rightarrow \quad d = 0.06542 \text{ or } 6.542\%$$

Alternatively, we have:

$$15{,}000(1+i)^3 = 18{,}375.65$$

$$\Rightarrow i = 0.07 \quad \Rightarrow \quad d = \frac{i}{1+i} = \frac{0.07}{1.07} = 0.06542 \text{ or } 6.542\%$$

Solution 1.11

The present value is:

$$500e^{-\delta t} = 500e^{-0.1\times 8} = \$224.66$$

Solution 1.12

The present value is:

$$1{,}000\exp\left[-\int_4^6 \delta_s\,ds\right] = 1{,}000\exp\left[-\int_4^6 (0.05+0.002s)\,ds\right] = 1{,}000\exp\left[-\left(0.05s+0.001s^2\right)\Big|_4^6\right]$$

$$= 1{,}000e^{-(0.336-0.216)} = 1{,}000e^{-0.12} = \$886.92$$

Solution 1.13

The accumulated value is:

$$30\exp\left[\int_0^5 \delta_s\,ds\right] = 30\exp\left[\int_0^5 (0.02s+0.01)\,ds\right] = 30\exp\left[\left(0.01s^2+0.01s\right)\Big|_0^5\right]$$

$$= 30e^{(0.3-0)} = 30e^{0.3} = \$40.50$$

Solution 1.14

The present value at time 10 years is:

$$1,000(1.04)^{-5}$$

So, the present value at time 5 years is:

$$1,000(1.04)^{-5}(1.09)^{-5}$$

Finally, the present value at time 0 is:

$$1,000(1.04)^{-5}(1.09)^{-5}(1.07)^{-5} = \$380.87$$

Solution 1.15

The accumulated value at time 5 years is:

$$100e^{5\times 0.07}$$

Hence, the accumulated value at time 8 years is:

$$100e^{5\times 0.07}e^{3\times 0.05} = \$164.87$$

Solution 1.16

If d_1 is the discount rate prior to 5 years ago and d_2 is the discount rate subsequently, then we have:

$$X(1-d_1)^{-3}(1-d_2)^{-5} = 457$$
$$\Rightarrow \quad X = 457(1-0.05)^3(1-0.04)^5$$
$$\Rightarrow \quad X = \$319.48$$

Solution 1.17

Both Bruce and Robbie earn the same rate of interest, and the interest Bruce earns during the 11th year is equal the interest Robbie earns in the 17th year. So Bruce's balance at the end of 10 years must be equal to Robbie's balance at the end of 16 years:

$$\frac{100}{(1-d)^{10}} = \frac{50}{(1-d)^{16}}$$
$$\Rightarrow (1-d)^6 = 0.5$$
$$\Rightarrow d = 0.1091$$

We can convert the discount rate to an interest rate:

$$i = \frac{d}{1-d} = \frac{0.1091}{1-0.1091} = 0.1225$$

The interest earned by Bruce in the 11th year equals the accumulated value of the fund at the end of the 10th year times the interest rate:

$$(100)(1.1225)^{10}(0.1225) = \$38.90$$

Solution 1.18

This force of interest question has two parts. The first part involves accumulating a deposit from time 0 to time 3. The second part involves determining the interest on the accumulated amount (including the new deposit of $\$X$ at time 3) from time 3 to time 6.

The accumulated value factor from time t_1 to time t_2 is:

$$\exp\left[\int_{t_1}^{t_2} 0.01t^2\, dt\right] = \exp\left[\left.\frac{t^3}{300}\right|_{t_1}^{t_2}\right] = \exp\left[\frac{(t_2)^3 - (t_1)^3}{300}\right]$$

Hence, the accumulated value at time 3 of the deposit of \$100 made at time 0 is:

$$100\exp\left[\frac{3^3 - 0^3}{300}\right] = 100e^{27/300} = 109.4174$$

This accumulated value at time 3, plus a deposit of $\$X$ at time 3, is accumulated from time 3 to time 6. This accumulated value is:

$$(109.4174 + X)\exp\left[\frac{6^3 - 3^3}{300}\right] = (109.4174 + X)e^{189/300} = 205.4433 + 1.877611X$$

The interest earned from time 3 to time 6 equals the accumulated value at time 6 less the accumulated value and the deposit at time 3. We are given that the interest earned over this period equals X. The interest earned from time 3 to time 6 is:

$$X = 205.4433 + 1.877611X - (109.4174 + X)$$
$$\Rightarrow X = \$784.60$$

Solution 1.19

The present value of the first payment stream is:

$$PV = 100 + 200v^n + 300v^{2n} = 100 + 200 \times 0.75941 + 300 \times 0.75941^2 = 424.893$$

The present value of the second payment stream is:

$$PV = 600v^{10}$$

We are given that the present values of the payment streams are equal. Hence:

$$600v^{10} = 424.893$$

$$\Rightarrow (1+i)^{-10} = \frac{424.893}{600}$$

$$\Rightarrow i = 0.03511 \text{ or } 3.511\%$$

Solution 1.20

First, we discount from time 10 to time 5 using the present value factor:

$$\exp\left[-\int_5^{10} \delta_t \, dt\right] = \exp\left[-\int_5^{10} 0.01(t^2 - t)\, dt\right] = \exp\left[-0.01\left(\frac{t^3}{3} - \frac{t^2}{2}\right)\Bigg|_5^{10}\right]$$

$$= \exp\left[-0.01\left(\frac{1{,}000}{3} - \frac{100}{2} - \frac{125}{3} + \frac{25}{2}\right)\right] = e^{-2.541667}$$

Next, we discount from time 5 to time 0 using the present value factor:

$$e^{-\delta t} = e^{-0.04\times 5} = e^{-0.2}$$

So, the present value of $100 payable at time 10 is:

$$100e^{-2.541667}e^{-0.2} = \$6.45$$

