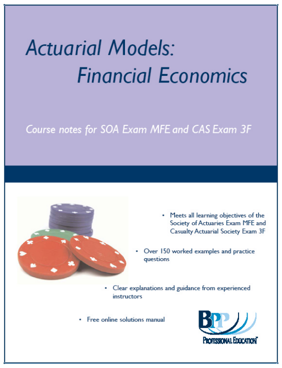

Financial economics (MFE)

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Solutions to practice questions – Chapter 1

Solution 1.1

Here we can use the "No dividends" version of the put-call parity relationship:

$$C_{Eur}(K,T)-P_{Eur}(K,T)=S_0-Ke^{-rT}$$

Remember that, for an at-the-money option, the underlying asset price equals the strike price, so $K=5$ here.

$$0.30-P_{Eur}(5,0.25)=5-5e^{-0.05\times0.25}$$

So: $P_{Eur}(5,0.25)=0.30-5+5e^{-0.05\times0.25}=\$0.24$

Solution 1.2

(a) We have been given a (non-zero) value for the parameter δ. So we must be assuming that the stock pays continuous dividends. The relevant put-call parity relationship is:

$$C_{Eur}(K,T)-P_{Eur}(K,T)=S_0e^{-\delta T}-Ke^{-rT}$$

Using the parameter values given, the left side of this "equation" is:

$$C_{Eur}(100,0.5)-P_{Eur}(100,0.5)=20-5=15$$

The right side is:

$$115e^{-0.03\times0.5}-100e^{-0.05\times0.5}=15.76$$

(b) Since the two sides of the "equation" are not exactly the same, the no-arbitrage principle has been violated. There is a potential risk-free profit to be made of \$0.76 ($=\$15.76-\$15$) from one call option and one put option, provided the costs of carrying out the transaction would not exceed the potential profit.

Solution 1.3

The next dividend is not due until after the option expires. So we can use the "No dividends" version of the put-call parity relationship:

$$C_{Eur}(K,T)-P_{Eur}(K,T)=S_0-Ke^{-rT}$$

Using the information given, the left side of this "equation" is:

$$C_{Eur}(2.50,0.5)-P_{Eur}(2.50,0.5)=0.35-0.10=0.25$$

The right side is:

$$2.75 - 2.50e^{-0.05\times0.5} = 0.31$$

Since the two sides of the "equation" are not exactly the same, there is a potential arbitrage opportunity.

In this case:

$$C_{Eur}(K,T) - P_{Eur}(K,T) < S_0 - Ke^{-rT}$$

We can rearrange this inequality as:

$$-S_0 + C_{Eur}(K,T) - P_{Eur}(K,T) + Ke^{-rT} < 0$$

The portfolio implied by the left side of this inequality "should" have a total value of zero, but in fact it has a negative value. This means that it would cost a negative amount to set up. In other words, setting up this portfolio would leave us with some money left over, which would be our arbitrage profit.

So what we need to do is:

- short sell 1 share (which will bring in \$2.75)
- buy 1 call option (which will cost \$0.35)
- sell 1 put option (which will bring in \$0.10)
- lend Ke^{-rT} in cash (which will cost \$2.44)

This will leave us with an initial arbitrage profit of $2.75 - 0.35 + 0.10 - 2.44 = \$0.06$ (which we can spend).

If we then wait till the maturity date and "close out" our positions, everything will cancel out. To check this:

- we will need to buy back the share (at an unknown cost of S_T)
- our call option will have a payoff of $\max(S_T - 2.50, 0)$
- our put option (a short position) will have a payoff of $-\max(2.50 - S_T, 0)$
- the loan will have accumulated with interest to K (=\$2.50)

So the final value of our position will be:

$$-S_T + \max(S_T - 2.50, 0) - \max(2.50 - S_T, 0) + 2.50$$

It is easy to check that this always equals zero whatever the value of S_T.

Solution 1.4

We can use the put-call parity relationship:

$$C_{Eur}(K,T) - P_{Eur}(K,T) = x_0 e^{-r_f T} - Ke^{-rT}$$

The parameter values here (working in units of dollars) are:

$$x_0 = 0.1, \quad K = 0.1, \quad r = 0.04, \quad r_f = 0.2, \quad T = 1$$

So: $$C_{Eur}(K,T) - 0.0165 = 0.1e^{-0.2} - 0.1e^{-0.04}$$

$$C_{Eur}(K,T) = 0.0165 + 0.1e^{-0.2} - 0.1e^{-0.04} = 0.0023$$

So the price of the corresponding call option is 0.23¢.

Solution 1.5

The prices will be the same if $C_{Eur}(K,T)-P_{Eur}(K,T)=0$. So the put call parity relationship tells us that:

$$0=S_0-PV_{0,T}(Div)-Ke^{-rT}$$

or $$e^{rT}\left[S_0-PV_{0,T}(Div)\right]=K$$

In other words, the forward price of the stock must equal the strike price.

Solution 1.6

We can rearrange this equation to make the call option the subject:

$$C_{Eur}(K,T)=P_{Eur}(K,T)+S_0e^{-\delta T}-Ke^{-rT}$$

We could synthesize one call option (a long position) by setting up the portfolio implied by the right side. To do this, we would need to:

- buy 1 European vanilla put option with strike price K and expiration time T
- buy $e^{-\delta T}$ shares
- borrow Ke^{-rT} in cash

We need to borrow cash here because we need to set up a portfolio where the cash component has a negative value.

We would not actually be able to buy a fractional number of shares ($e^{-\delta T}$ is less than 1). But in practice, we would be synthesizing a large number of call options, so we could round this to the nearest whole number.

Solution 1.7

(a) We know that, for put options:

$$\max[0,PV_{0,T}(K)-PV_{0,T}(F_{0,T})] \le P_{Eur}(S,K,T) \le P_{Amer}(S,K,T) \le K$$

Here we have an American option and we can say that:

$$\max[0,PV_{0,T}(10)-PV_{0,T}(F_{0,T})] \le P_{Amer}(S,10,T) \le 10$$

Since we are not told the current price of the underlying asset, the left side could be as low as zero. So all we can be sure of from this is that:

$$0\le P_{Amer}(S,10,T)\le 10$$

So we can only conclude that the price of the strike-10 put option lies somewhere between 0 and 10.

(b) Here we can apply Theorem 1.1 with $K_1=10$ and $K_2=15$, which tells us that:

$$P(K_2)\ge P(K_1)$$

So: $$P(15)\ge P(10)=2$$

We can also apply Theorem 1.2, which tells us that:

$$P(K_2)-P(K_1)\le K_2-K_1$$

So: $$P(15)-P(10)\le 15-10$$

$$P(15)\le P(10)+15-10=2+15-10=7$$

So we can conclude that the price of the strike-15 put option lies somewhere between 2 and 7.

(c) Here we can apply Theorem 1.3 with $K_1 = 10$, $K_2 = 15$ and $K_3 = 20$, which tells us that:

$$\frac{P(K_2) - P(K_1)}{K_2 - K_1} \le \frac{P(K_3) - P(K_2)}{K_3 - K_2}$$

So:

$$\frac{P(15) - P(10)}{15 - 10} \le \frac{P(20) - P(15)}{20 - 15}$$

$$\frac{P(15) - 2}{5} \le \frac{6 - P(15)}{5}$$

Multiplying through by 5:

$$P(15) - 2 \le 6 - P(15)$$

$$2P(15) \le 8$$

$$P(15) \le 4$$

So we can conclude that the price of the strike-15 put option lies somewhere between 2 and 4.

Solution 1.8

The proof for Theorem 1.3, with $K_1 = 80$, $K_2 = 100$ and $K_3 = 125$ tells us that the required portfolio to create an arbitrage. Here:

$$\lambda = \frac{125 - 100}{125 - 80} = \frac{5}{9}$$

To avoid the fractions, we can scale everything in the portfolio up by a factor of 9. So to create the arbitrage, we could:

- Purchase 5 of the 80-strike options
- Sell 9 of the 100-strike options
- Purchase 4 of the 125-strike options

Using the prices given, the cash flows involved in setting up this portfolio would be:

$$-5 \times 30 + 9 \times 25 - 4 \times 15 = +15$$

So this would generate an initial arbitrage profit of 15.

If we then do nothing until the expiration date, the total payoff from this portfolio will be:

$$5\max(0, S_T - 80) - 9\max(0, S_T - 100) + 4\max(0, S_T - 125)$$

We can check that this always works out to zero or a higher amount, by considering the various cases:

$125 < S_T$: $5(S_T - 80) - 9(S_T - 100) + 4(S_T - 125) = 0$
$100 < S_T \le 125$: $5(S_T - 80) - 9(S_T - 100) + 4(0) = -4S_T + 500 = 4(125 - S_T) > 0$
$80 < S_T \le 100$: $5(S_T - 80) - 9(0) + 4(0) = 5(S_T - 80) > 0$
$S_T \le 80$: $5(0) - 9(0) + 4(0) = 0$

So this portfolio generates an initial profit of 15 and a final payoff of at least zero. So we have a risk-free profit.

Solution 1.9

The graph will be convex, because the sum of convex functions is also convex.

Solution 1.10

We can use the formula:

$$P_\$(x_0, K, T) = x_0 K C_f(x_0^{-1}, K^{-1}, T)$$

Here we have:

$x_0 = ?$, $K = 1$ (the value of T is not needed)

So: $0.33 = x_0 \times 1 \times 0.5$

$x_0 = 0.66$

So 1 unit of the foreign currency is equivalent to \$0.66.
