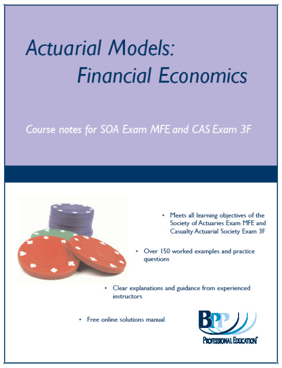

PROFESSIONAL EDUCATION™

Financial economics (MFE)

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Solutions to practice questions – Chapter 4

Solution 4.1

We now have:

$$S = 41\,,\ K = 45\,,\ \sigma = 0.25\,,\ r = 0.05\,,\ T = 1\,,\ \delta = 0.04$$

The values of d_1 and d_2 are:

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - \delta + \tfrac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} = \frac{\ln\left(\frac{41}{45}\right) + \left(0.05 - 0.04 + \tfrac{1}{2}\times 0.25^2\right)(1)}{0.25\sqrt{1}} = -0.2074$$

and $\quad d_2 = d_1 - \sigma\sqrt{T} = -0.2074 - 0.25\sqrt{1} = -0.4574$

So the price of the call option is:

$$\begin{aligned} C_{Eur} &= Se^{-\delta T}N(d_1) - Ke^{-rT}N(d_2) \\ &= 41e^{-0.04}N(-0.21) - 45e^{-0.05}N(-0.46) \\ &= 39.39 \times 0.4168 - 42.81 \times 0.3228 = 2.60 \end{aligned}$$

Solution 4.2

Delta measures the sensitivity of the option price to a small change in the underlying price S. Here we have increased S by 1 (with no change in the other parameters). So we should have:

$$\text{Change in option price} \approx \varepsilon \times \Delta = 1 \times 0.3635 = 0.3635$$

So the new option price should be approximately:

$$\text{New option price} \approx 2.22 + 0.3635 = 2.5835$$

This is quite close to our calculated value of 2.60. So our calculations seem reasonable.

Solution 4.3

Vega measures the sensitivity of the option price to a small change in the volatility σ. Here we have increased σ by 2 percentage points (with no change in the other parameters). So we should have:

$$\text{Change in option price} \approx \varepsilon \times Vega = 2 \times 0.1463 = 0.2926$$

So the new option price should be approximately:

$$\text{New option price} \approx 2.22 + 0.2926 = 2.5126$$

If you recalculate the option price accurately, you will get 2.52. So again this gives a good approximation.

Solution 4.4

1. The delta for a call option is POSITIVE whereas the delta for a put option is NEGATIVE.
2. Gamma measures the sensitivity of DELTA to small changes in the UNDERLYING ASSET (STOCK) PRICE. It is POSITIVE for both call and put options.
3. Vega measures the sensitivity of the OPTION PRICE to small changes in the VOLATILITY. It is POSITIVE for call options and POSITIVE for put options.
4. Option values usually decrease as time passes, even when the other parameter values remain unchanged. As a result, the Greek THETA usually has a NEGATIVE value.
5. The names of the other two Greeks are RHO and PSI.

Solution 4.5

The Black-Scholes formulas are derived by calculating the discounted expected value of the payoff at the expiration date T. They make no allowance for other contingencies before that time, such as the possibility of early exercise. So they cannot deal with American options.

Solution 4.6

The general form of the Black-Scholes formula for a put option is:

$$P_{Eur} = F_{0,T}^P(K)N(-d_2) - F_{0,T}^P(S)N(-d_1)$$

where $d_1 = \dfrac{\ln\left(\dfrac{F_{0,T}^P(S)}{F_{0,T}^P(K)}\right) + \dfrac{\sigma^2 T}{2}}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$.

Here we have:

$$S = 70\,,\ K = 75\,,\ \sigma = 0.2\,,\ r = 0.05\,,\ T = 0.25$$

Since the underlying asset is a future, the prepaid forward prices are:

$$F_{0,T}^P(S) = Fe^{-rT} = 70e^{-0.05(0.25)} = 69.13$$

and $F_{0,T}^P(K) = Ke^{-rT} = 75e^{-0.05(0.25)} = 74.07$

So: $$d_1 = \frac{\ln\left(\frac{69.13}{74.07}\right) + \frac{1}{2}(0.2)^2(0.25)}{0.2\sqrt{0.25}} = -0.6399$$

and $$d_2 = -0.6399 - 0.2\sqrt{0.25} = -0.7399$$

The price of the put option is then:

$$\begin{aligned} P_{Eur} &= 74.07 \times N(0.74) - 69.13 \times N(0.64) \\ &= 74.07 \times 0.7703 - 69.13 \times 0.7389 \\ &= 5.98 \end{aligned}$$

Solution 4.7

The market price of 10.50 lies between 9.80 and 10.95. So the implied volatility lies somewhere between 15% and 20%. We can estimate it using linear interpolation:

$$\sigma \approx 15\% + \left(\frac{10.50 - 9.80}{10.95 - 9.80}\right) \times (20\% - 15\%) = 18.0\%$$

Solution 4.8

The completed table looks like this:

Component	Number of units held	Current value	Delta	Gamma	Elasticity
Underlying asset	65,676	\$0.7200	1	0	1
Call options	–100,000	\$0.0263	0.37176	0.034	10.2
Put options	50,000	\$0.0616	–0.57000	0.034	–6.7
Portfolio		\$47,737	0	–1,700	0

Delta is the sensitivity of a security price to small changes in the underlying asset price. So, for the underlying asset, itself, delta must have a constant value of 1.

Gamma is the sensitivity of delta to small changes in the underlying asset price. So, for the underlying asset, itself, gamma must equal 0 (since delta is constant).

We can find delta for the call option from the relationship:

$$\Delta_{put} = \Delta_{call} - e^{-\delta T}$$

Here we have $T = 1$ and $\delta = 0.06$ (the interest rate on the foreign currency).

So: $$\Delta_{call} = \Delta_{put} + e^{-\delta T} = -0.57 + e^{-0.06} = 0.37176$$

Gamma for the put options is the same as gamma for the call options.

We can use the fact that the overall delta is zero to find the size of the holding of the underlying asset ($= x$, say):

$$x \times 1 - 100{,}000 \times 0.37176 + 50{,}000 \times (-0.57) = 0 \qquad \Rightarrow x = 65{,}676$$

We can then find the overall value of the portfolio:

$$\textit{Portfolio value} = 65,676 \times 0.72 - 100,000 \times 0.0263 + 50,000 \times 0.0616 = \$47,737$$

We also need to find the overall gamma of the portfolio:

$$\Gamma_{portfolio} = 65,676 \times 0 - 100,000 \times 0.034 + 50,000 \times 0.034 = -1,700$$

The elasticities can be calculated from the formula $\Omega = \frac{\Delta S}{f}$:

$$\Omega_{underlying} = \frac{\Delta_{underlying} \times S}{f_{underlying}} = \frac{1 \times 0.7200}{0.7200} = 1$$

$$\Omega_{call} = \frac{\Delta_{call} \times S}{f_{call}} = \frac{0.37176 \times 0.7200}{0.0263} = 10.2$$

$$\Omega_{put} = \frac{\Delta_{put} \times S}{f_{put}} = \frac{-0.57000 \times 0.7200}{0.0616} = -6.7$$

Because the overall delta for the trader's portfolio is zero, its elasticity is also zero, *ie* $\Omega_{portfolio} = 0$.

