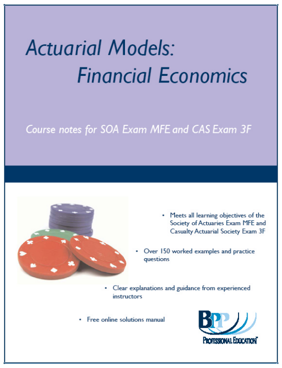


BPP PROFESSIONAL EDUCATION™

# Financial economics (MFE)

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# *Solutions to practice questions – Chapter 9*

## *Solution 9.1*

The distribution function for this distribution is:

$$F_X(x) = \Pr(X \le x) = \int_0^x 0.01e^{-0.01t}\,dt = 1 - e^{-0.01x}\,, \quad x > 0$$

To find the inverse function, we can let $u = F_X(x) = 1 - e^{-0.01x}$ and rearrange to make $x$ the subject of the equation:

$$x = -100\ln(1-u) \quad \Rightarrow \quad x = F_X^{-1}(u) = -100\ln(1-u)$$

If we now substitute the value of the pseudorandom number $u_1 = 0.49$, we get:

$$x = F_X^{-1}(u_1) = -100\ln(1-u_1) = -100\ln(1-0.49) = 67$$

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## *Solution 9.2*

The distribution function for this distribution is:

$$F_X(x) = \Pr(X \le x) = N(x) \text{ from the Tables}$$

We can't invert this function algebraically, but from the Tables we find that:

$$F_X(-0.61) = N(-0.61) = 1 - N(0.61) = 1 - 0.7291 = 0.2709 \approx u_2$$

So the required value is:

$$x = F_X^{-1}(u_2) = F_X^{-1}(0.27) = -0.61$$

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## *Solution 9.3*

The distribution function for this distribution is:

$$F_X(x) = \Pr(X \le x) = \int_0^x 6t(1-t)dt = \int_0^x (6t - 6t^2)dt = \Big[3t^2 - 2t^3\Big]_0^x = 3x^2 - 2x^3, \quad 0 < x < 1$$

To find a formula for the inverse function, we would need to invert the equation $u = F_X(x) = 3x^2 - 2x^3$. This would involve solving a cubic equation, which we cannot do easily. So we will need to find the inverse value corresponding to $u_3 = 0.47$ numerically. If we try a few values, we can quickly home in on the correct answer:

Try $x = 0.5$: $F_X(0.5) = 3(0.5)^2 - 2(0.5)^3 = 0.5 > 0.47$

Try $x = 0.4$: $F_X(0.4) = 3(0.4)^2 - 2(0.4)^3 = 0.352 < 0.47$

Try $x = 0.48$: $F_X(0.48) = 3(0.48)^2 - 2(0.48)^3 = 0.4700$ ✓

So the required value is:

$$x = F_X^{-1}(u_3) = F_X^{-1}(0.47) = 0.48$$

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## *Solution 9.4*

The distribution function for this discrete distribution is:

$$F_X(0) = \Pr(X \le 0) = \Pr(X = 0) = e^{-4} = 0.0183$$

$$F_X(1) = \Pr(X \le 1) = \Pr(X \le 0) + \Pr(X = 1) = 0.0183 + 4e^{-4} = 0.0183 + 0.0733 = 0.0916$$

$$F_X(2) = \Pr(X \le 2) = \Pr(X \le 1) + \Pr(X = 2) = 0.0916 + \frac{4^2}{2!}e^{-4} = 0.0916 + 0.1465 = 0.2381 \ \textit{etc}$$

To find the inverse value corresponding to $u_4 = 0.11$, we note that $F_X(1) < u_4 < F_X(2)$, so that:

$$x = F_X^{-1}(u_4) = F_X^{-1}(0.11) = 2$$

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## *Solution 9.5*

The distribution function for this discrete distribution is:

$$F_X(0) = \Pr(X \le 0) = \Pr(X = 0) = (0.5)^4 = 0.0625$$

$$F_X(1) = \Pr(X \le 1) = \Pr(X \le 0) + \Pr(X = 1) = 0.0625 + \binom{4}{1}(0.5)^3(0.5) = 0.0625 + 0.25 = 0.3125$$

$$F_X(2) = \Pr(X \le 2) = \Pr(X \le 1) + \Pr(X = 2) = 0.3125 + \binom{4}{2}(0.5)^2(0.5)^2 = 0.3125 + 0.375 = 0.6875 \ \textit{etc}$$

To find the inverse value corresponding to $u_5 = 0.52$, we note that $F_X(1) < u_5 < F_X(2)$, so that:

$$x = F_X^{-1}(u_5) = F_X^{-1}(0.52) = 2$$

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## *Solution 9.6*

The mean and variance of the suggested formula for $Z$ are:

$$E[Z]=E\left[\sum_{i=1}^{m}U_i-n\right]=\tfrac{1}{2}m-n \quad \text{and} \quad \operatorname{var}[Z]=\operatorname{var}\left[\sum_{i=1}^{m}U_i-n\right]=\tfrac{1}{12}m$$

If we equate these to 0 and 1 to match a standard normal distribution, we find that the only solution is $m=12$ and $n=6$, *ie* $Z=\sum_{i=1}^{12}U_i-6$. So this is the only formula of this form that works.

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## *Solution 9.7*

Using the random numbers $u_6=0.92$ and $u_7=0.24$ in the Box-Muller formula, we get:

$$Z_1=\sqrt{-2\ln u_6}\cos(2\pi u_7)=\sqrt{-2\ln 0.92}\cos(2\pi\times 0.24)=0.41\times 0.06=0.03$$

and $$Z_2=\sqrt{-2\ln u_6}\sin(2\pi u_7)=\sqrt{-2\ln 0.92}\sin(2\pi\times 0.24)=0.41\times 1.00=0.41\,.$$

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## *Solution 9.8*

If $Z\sim N(0,1)$, then $2+0.5Z\sim N(2,0.25)$ and $\exp(2+0.5Z)\sim LogN(2,0.25)$. So, using the pseudorandom numbers $z_1=-0.82$ and $z_2=1.44$ (from the standard normal distribution), we get:

$$\exp(2+0.5z_1)=\exp\left[2+0.5(-0.82)\right]=4.90$$

and $$\exp(2+0.5z_2)=\exp\left[2+0.5(1.44)\right]=15.18\,.$$

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## *Solution 9.9*

For a single path of Brownian motion, the values of $Z(10)$ and $Z(20)$ will not be independent. So we need to simulate values for the increments $Z(10)-Z(0)$ and $Z(20)-Z(10)$. These are independent and each has a $N(0,10)$ distribution. The simulated values of the increments are:

$$\sqrt{10}z_3=\sqrt{10}\times 0.02=0.06$$

and $$\sqrt{10}z_4=\sqrt{10}\times -0.29=-0.92$$

These correspond to values of $Z(10)=0.06$ and $Z(20)=\{Z(10)-Z(0)\}+\{Z(20)-Z(10)\}=0.06+(-0.92)=-0.86$.

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## *Solution 9.10*

If $X\sim LogN(\mu,\sigma^2)$, then $\ln X\sim N(\mu,\sigma^2)$. So we can calculate the sample mean and sample variance of the data values $\ln x_1,\ln x_2,\ldots,\ln x_n$ and use these as estimates of $\mu$ and $\sigma^2$.

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## *Solution 9.11*

(a) To find $\Pr(X>1)$, we can use the fact that $\ln X \sim N(0,1)$:

$$\Pr(X>1)=\Pr(\ln X>0)=\Pr\left[N(0,1)>0\right]=0.5$$

(b) To find $E(X)$, we can use the formula for the mean of a lognormal distribution:

$$E(X)=e^{\mu+½\sigma^2}=e^{0+½(1)^2}=e^{0.5}=1.65$$

(c) To find $E(X^2)$, we can use the general formula $E(X^r)=\exp(r\mu+½r^2\sigma^2)$ with $r=2$:

$$E(X^2)=\exp(2\mu+2\sigma^2)=e^2=7.39$$

(d) To find $\text{var}(X)$, we can use the previous two results:

$$\text{var}(X)=E(X^2)-\left[E(X)\right]^2=e^2-(e^{0.5})^2=e^2-e=4.67$$

(e) To find $E(\sqrt{X})$, we can use the general formula $E(X^r)=\exp(r\mu+½r^2\sigma^2)$ with $r=½$:

$$E(\sqrt{X})=\exp(0.5\mu+0.125\sigma^2)=e^{0.125}=1.13$$

(f) To find $E(1/X)$, we can use the general formula $E(X^r)=\exp(r\mu+½r^2\sigma^2)$ with $r=-1$:

$$E(1/X)=\exp(-\mu+0.5\sigma^2)=e^{0.5}=1.65$$

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## *Solution 9.12*

If the quantity we are estimating is $E(X)$, we are given that:

$$2\times1.96\frac{\hat{\sigma}_X}{\sqrt{100}}=0.1E(X) \quad \Rightarrow \frac{\hat{\sigma}_X}{E(X)}=\frac{0.1}{2\times1.96}\sqrt{100}$$

From the Tables we can see that the upper 0.5th percentile of the standard normal distribution is between 2.57 and 2.58. So, if we round up to be on the safe side, we need to find the value of $n$ such that:

$$2\times2.58\frac{\hat{\sigma}_X}{\sqrt{n}}<0.01E(X)$$

So: $$\sqrt{n}>\frac{2\times2.58}{0.01}\frac{\hat{\sigma}_X}{E(X)}=\frac{2\times2.58}{0.01}\times\frac{0.1}{2\times1.96}\sqrt{100}=132 \quad \Rightarrow n>17,300$$

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