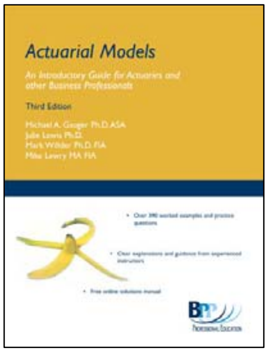

Actuarial Models

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By M Gauger, J Lewis, M Willder, M Lewry

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Solutions to practice questions – Chapter 11

Solution 11.1

The development of the contract fund over the first three months is as follows:

Policy month	Premium	Expense charge	Mortality charge	Interest	Investment charge	Ending fund
1	500	300	149.80	0.20	0.10	50.30
2	500	5	149.45	1.61	0.79	396.67
3	500	5	149.11	3.03	1.49	744.09

The contract fund at the end of each time period is given by:

$$V_t = \left(V_{t-1} + 500 - EC_t - MC_t\right)(1.05)^{1/12} \times (1 - 0.002)$$

The expense charge (EC_t) is 300 in the first month and 5 thereafter.

The mortality charge is given by:

$$MC_t = \left[150{,}000 - \left(V_{t-1} + 500 - EC_t\right)\right] \times 0.001$$

Solution 11.2

The development of the contract fund over the first three years is as follows:

Policy year	Premium	Expense charge	Mortality charge	Interest	Investment charge	Ending fund
1	2,000	25	570.38	70.23	7.37	1,467.48
2	2,000	25	548.36	173.65	15.34	3,052.43
3	2,000	25	524.59	315.20	24.09	4,793.95

The contract fund at the end of each time period is given by:

$$V_t = \left(V_{t-1} + 500 - 25 - MC_t\right)(1+i) \times (1-0.005)$$

The interest rate is 5%, 6% and 7% in the first, second and third years respectively.

The mortality charge is given by:

$$MC_t = \left[40,000 - \left(V_{t-1} + 2,000 - 25\right)\right] \times 0.015$$

Solution 11.3

Surrender charge at end of year $1 = 1,467.48197 \times 0.2 = 293.49639$
Surrender charge at end of year $2 = 3,052.42752 \times 0.09 = 274.71848$
Surrender charge at end of year $3 = 4,793.94747 \times 0.055 = 263.66711$

Cash surrender value at end of year $1 = 1,467.48197 \times (1-0.2) = 1,173.98558$
Cash surrender value at end of year $2 = 3,052.42752 \times (1-0.09) = 2,777.70904$
Cash surrender value at end of year $3 = 4,793.94747 \times (1-0.055) = 4,530.28036$

Solution 11.4

For the policyholder to continue to pay premiums for three years the policyholder must remain in the premium paying state (state 0) for three years. Hence the probability is as follows:

$${}_3P^{(0)} = \left(Q^{(0,0)}\right)^3 = 0.84^3 = 0.59270$$

For the policyholder to be paying premiums in three years' time, the policyholder must either have remained in state 0 for three years or have moved into the premium cessation state (state 4) and returned to state 0. Hence the probability is as follows:

$$\begin{aligned}
{}_3Q^{(0,0)} &= \left(Q^{(0,0)}\right)^3 + Q^{(0,0)} \times Q^{(0,4)} \times Q^{(4,0)} + Q^{(0,4)} \times Q^{(4,4)} \times Q^{(4,0)} + Q^{(0,4)} \times Q^{(4,0)} \times Q^{(0,0)} \\
&= 0.84^3 + 0.84 \times 0.05 \times 0.09 + 0.05 \times 0.7 \times 0.09 + 0.05 \times 0.09 \times 0.84 \\
&= 0.60341
\end{aligned}$$

The policyholder can surrender over the next two years from the premium paying state in year 1 or from either of the active states in year 2. Hence the probability is as follows:

$$\begin{aligned}
{}_2Q^{(0,3)} &= Q^{(0,3)} + Q^{(0,0)} \times Q^{(0,3)} + Q^{(0,4)} \times Q^{(4,3)} \\
&= 0.1 + 0.84 \times 0.1 + 0.05 \times 0.2 \\
&= 0.19400
\end{aligned}$$

Solution 11.5

The gross premium is calculated as follows:

$$\sum_{t=1}^{\infty} GPv^{t-1} \frac{l_{40+t-1}}{l_{40}} = \sum_{t=1}^{\infty} \left(50,000v^t \frac{d_{40+t-1}}{l_{40}} \right) + \sum_{t=1}^{\infty} \left(E_t v^{t-1} \right) \frac{l_{40+t-1}}{l_{40}}$$

$$GP\ddot{a}_{40} = 50,000A_{40} + 250 + 30\ddot{a}_{40}$$

$$GP \times 19.4 = 50,000 \times 0.25 + 250 + 30 \times 19.4$$

$$GP = 687.21649$$

Hence the guarantee fund at time 10 is given by:

$$GF_{10} = \sum_{t=11}^{\infty} \left(50,000v^{t-10} \frac{d_{40+t-1}}{l_{50}} \right) + \sum_{t=11}^{\infty} \left(E_t v^{t-11} \right) \frac{l_{40+t-1}}{l_{50}} - \sum_{t=11}^{\infty} GPv^{t-11} \frac{l_{40+t-1}}{l_{50}}$$

$$= 50,000A_{50} + 30\ddot{a}_{50} - GP\,\ddot{a}_{50}$$

$$= 50,000 \times 0.35 + 30 \times 17.1 - 687.21649 \times 17.1$$

$$= 6,261.59794$$

As the guarantee fund is smaller than the contract fund value of 6,300, the insurer holds a reserve of 6,300.