Probability, Fourth Edition

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Solutions to practice questions – Chapter 3

Solution 3.1

We require $\Pr(B)$ and $\Pr(A \cap B)$. We have:

$$\begin{aligned}\Pr(A \cup B) &= \Pr(A) + \Pr(B) - \Pr(A \cap B) = \Pr(A) + 4\Pr(A \cap B) - \Pr(A \cap B)\\ &= \Pr(A) + 3\Pr(A \cap B)\end{aligned}$$

$$\Rightarrow \quad 0.75 = 0.39 + 3\Pr(A \cap B)$$

$$\Rightarrow \quad \Pr(A \cap B) = 0.12\,, \quad \Pr(B) = 0.48$$

From the definition of conditional probability:

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)} = \frac{0.12}{0.48} = 0.25$$

Solution 3.2

We have:

$$\Pr(B|A') = \frac{\Pr(A' \cap B)}{\Pr(A')} = \frac{\Pr(A')\Pr(B)}{\Pr(A')} = \Pr(B) = 0.3$$

Intuitively, since A and B are independent, $\Pr(B|A') = \Pr(B)$.

Solution 3.3

Let A be the event that the first student selected is a man, and let B be the event that the second student selected is a man. Then:

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)} = \frac{\Pr(A)\Pr(B|A)}{\Pr(A)\Pr(B|A) + \Pr(A')\Pr(B|A')}$$

$$= \frac{(6/10)(5/9)}{(6/10)(5/9) + (4/10)(6/9)} = \frac{5}{9}$$

Note: Intuitively, if we know that the second student selected is a man, the probability that the first student selected is a man is 5 (the number of men remaining) divided by 9 (the number of students remaining).

Solution 3.4

Let N represent nausea and let V represent impaired vision.

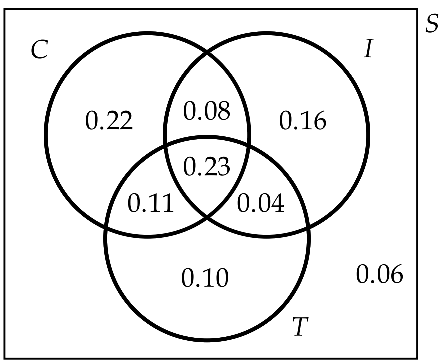
Then we have:

$$\Pr(V|N') = \frac{\Pr(V \cap N')}{\Pr(N')} = \frac{\Pr(V) - \Pr(V \cap N)}{\Pr(N')}$$

$$= \frac{0.63 - 0.32}{1 - 0.59} = 0.7561$$

Solution 3.5

The probabilities are shown in the following Venn diagram:



Hence we have:

$$\Pr((I' \cap T')|C) = \frac{\Pr(C \cap (I' \cap T'))}{\Pr(C)} = \frac{0.22}{0.64} = 0.34375$$

Solution 3.6

If 60% of the policies are for male lives, then 40% are for female lives. Of the policies for male lives, 15% have sums assured in excess of \$500,000. Hence in absolute terms, $60\% \times 15\% = 9\%$ of all policies are for male lives and with sums assured in excess of \$500,000, and $60\% - 9\% = 51\%$ are for male lives and with sums assured below \$500,000. Using a similar method, the full breakdown of the policies is as follows:

	Male life	**Female life**	**Total**
Less than \$500,000	51%	37.6%	**88.6%**
Exceeds \$500,000	9%	2.4%	**11.4%**
Total	**60%**	**40%**	**100%**

Hence the required probability is:

$$\Pr(\text{Female}|\text{Less than \$500,000}) = \frac{\Pr(\text{Female} \cap \text{Less than \$500,000})}{\Pr(\text{Less than \$500,000})} = \frac{0.376}{0.886} = 0.4244$$

Solution 3.7

Let A_i $(i=1,2,3,4,5,6)$ be the event that the score on the first die is i. Then:

$$\Pr(A_i)=\frac{1}{6} \qquad \text{for } i=1,2,3,4,5,6$$

Let B be the event that the game ends when the score on the second die is a 3.

If the score on the first die is a 1 or a 2, then event B is impossible, *ie*:

$$\Pr(B|A_1)=0 \qquad \Pr(B|A_2)=0$$

If the score on the first die is a 3, the game ends when a 1, 2, or 3 is rolled. Hence, we have:

$$\Pr(B|A_3)=\frac{1}{3}$$

Similarly, the other conditional probabilities are:

$$\Pr(B|A_4)=\frac{1}{4} \qquad \Pr(B|A_5)=\frac{1}{5} \qquad \Pr(B|A_6)=\frac{1}{6}$$

Using the law of total probability, we have:

$$\begin{aligned}\Pr(B)&=\sum_{i=1}^{6}\Pr(A_i)\Pr(B|A_i)\\&=\frac{1}{6}\times\left(0+0+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}\right)=\frac{19}{120}=0.1583\end{aligned}$$

Solution 3.8

Let A_i $(i=0,1,2)$ be the event that the two balls drawn from the first urn include i red balls. Then:

$$\Pr(A_0)=\frac{{}_{12}C_2}{{}_{22}C_2}=0.2857$$

$$\Pr(A_1)=\frac{({}_{12}C_1)({}_{10}C_1)}{{}_{22}C_2}=0.5195$$

$$\Pr(A_2)=\frac{{}_{10}C_2}{{}_{22}C_2}=0.1948$$

Let B be the event that the ball drawn from the second urn is red. Since all red balls will be transferred to the second urn, the conditional probabilities are:

$$\Pr(B|A_i)=\frac{6+i}{10+i} \qquad \text{for } i=0,1,2$$

Using the law of total probability, we have:

$$\begin{aligned}\Pr(B)&=\Pr(A_0)\Pr(B|A_0)+\Pr(A_1)\Pr(B|A_1)+\Pr(A_2)\Pr(B|A_2)\\&=0.2857\times\frac{6}{10}+0.5195\times\frac{7}{11}+0.1948\times\frac{8}{12}=0.6319\end{aligned}$$

Solution 3.9

We can break down the data in the question into the following table:

	Twins	**Not twins**	**Total**
Grandparent was twin	10	164	**174**
Grandparent was not twin	51	1,670	**1,721**
Total	**61**	**1,834**	**1,895**

We can divide each of these numbers by 1,895 to give us the probabilities (and note that this divisor will cancel from both terms in the formula), hence:

$$\Pr\left(\text{Twins}\middle|\text{Grandparent not twin}\right)=\frac{\Pr\left(\text{Twins}\cap\text{Grandparent not twin}\right)}{\Pr\left(\text{Grandparent not twin}\right)}$$

$$=\frac{51/1,895}{1,721/1,895}=\frac{51}{1,721}=0.0296$$

Solution 3.10

Let x be the number of automobile policyholders and let y be the number of homeowners policyholders.

The total number of policyholders (avoiding double counting) is:

$$x+y-88=1,000$$

We know that there are three times as many policyholders with automobile insurance than policyholders with homeowners insurance, so:

$$x=3y$$
$$\Rightarrow 4y=1,088$$
$$\Rightarrow y=272,\ x=816$$

Using the definition of conditional probability:

$$\Pr\left(\text{No homeowners}\middle|\text{Automobile}\right)=\frac{\Pr\left(\text{No homeowners}\cap\text{Automobile}\right)}{\Pr\left(\text{Automobile}\right)}$$

$$=\frac{(1,000-272)/1,000}{816/1,000}=\frac{0.728}{0.816}=0.8922$$

Solution 3.11

Let C , Se , and St denote critical, serious, and stable respectively. Let D denote the patient dying.

Using Bayes' Theorem:

$$\Pr(Se|D') = \frac{\Pr(Se \cap D')}{\Pr(D')}$$

$$= \frac{\Pr(Se)\Pr(D'|Se)}{\Pr(C)\Pr(D'|C) + \Pr(Se)\Pr(D'|Se) + \Pr(St)\Pr(D'|St)}$$

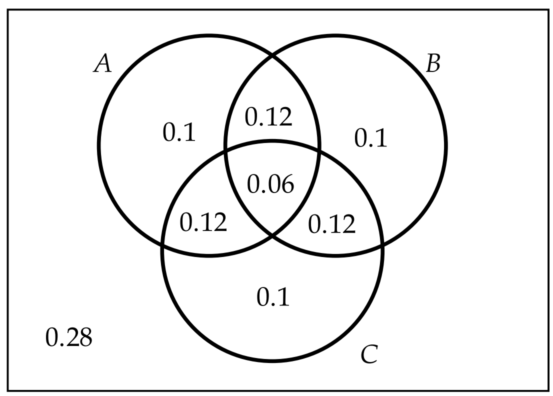
$$= \frac{0.30(1-0.10)}{0.10(1-0.40) + 0.30(1-0.10) + 0.60(1-0.01)}$$

$$= \frac{0.27}{0.924} = 0.2922$$

Solution 3.12

Let's solve this problem with the help of a Venn diagram. We can quickly fill in the values of 0.1 and 0.12 for the areas of the graph relating to exactly one and two risk factors respectively.

We can calculate $\Pr(A \cap B \cap C)$ using the relationship:



$$\Pr(C|A \cap B) = \frac{\Pr(A \cap B \cap C)}{\Pr(A \cap B)}$$

$$\Rightarrow \quad \frac{1}{3} = \frac{\Pr(A \cap B \cap C)}{0.12 + \Pr(A \cap B \cap C)}$$

$$\Rightarrow \quad \Pr(A \cap B \cap C) = 0.06$$

$$\Rightarrow \quad \Pr(A' \cap B' \cap C') = 0.28$$

Finally, we have:

$$\Pr\left((B \cup C)'|A'\right) = \Pr(B' \cap C'|A')$$

$$= \frac{\Pr(A' \cap B' \cap C')}{\Pr(A')}$$

$$= \frac{0.28}{1 - 0.1 - 0.12 - 0.12 - 0.06} = \frac{0.28}{0.60} = 0.4667$$

Solution 3.13

The probabilities are summarized in the following table:

Risk class (A_i)	$\Pr(A_i)$	$\Pr(\text{No claims} \mid A_i)$
Low	0.60	0.80
Moderate	0.25	0.60
High	0.15	0.40

Using Bayes' Theorem:

$$\Pr(M|N) = \frac{\Pr(M \cap N)}{\Pr(N)}$$

$$= \frac{\Pr(M)\Pr(N|M)}{\Pr(L)\Pr(N|L) + \Pr(M)\Pr(N|M) + \Pr(H)\Pr(N|H)}$$

$$= \frac{0.25 \times 0.60}{0.60 \times 0.80 + 0.25 \times 0.60 + 0.15 \times 0.40}$$

$$= 0.2174$$

Solution 3.14

Let S stand for smoker and let C stand for circulatory problem. We need to calculate $\Pr(C|S)$.

We are given:

$$\Pr(C) = 0.25$$
$$\Pr(S|C) = 2\Pr(S|C')$$

From the definition of conditional probability:

$$\Pr(C|S) = \frac{\Pr(C \cap S)}{\Pr(S)} = \frac{\Pr(C)\Pr(S|C)}{\Pr(S)} = \frac{0.25 \times \Pr(S|C)}{\Pr(S)}$$

$$\Pr(C'|S) = \frac{\Pr(C' \cap S)}{\Pr(S)} = \frac{\Pr(C')\Pr(S|C')}{\Pr(S)} = \frac{0.75 \times (0.5 \times \Pr(S|C))}{\Pr(S)} = \frac{0.375 \times \Pr(S|C)}{\Pr(S)}$$

$$\Rightarrow \Pr(C'|S) = 1.5 \times \Pr(C|S)$$

These two conditional probabilities must sum to 1, so:

$$\Pr(C|S) = 1 - \Pr(C'|S) = 1 - 1.5 \times \Pr(C|S)$$

$$\Rightarrow \Pr(C|S) = \frac{2}{5} = 0.4$$

Solution 3.15

Since we are given that the car was from one of the model years 1997, 1998, or 1999, we need to remove the category "other" from the data. Since 54% of the cars are from model years 1997, 1998, and 1999, the revised proportions are:

$$1997: \quad 0.16/0.54$$
$$1998: \quad 0.18/0.54$$
$$1999: \quad 0.20/0.54$$

Let A denote an accident from one of the model years 1997, 1998 or 1999.

Applying Bayes' Theorem:

$$\begin{aligned}
\Pr(1997|A) &= \frac{\Pr(1997 \cap A)}{\Pr(A)} \\
&= \frac{\Pr(1997)\Pr(A|1997)}{\Pr(1997)\Pr(A|1997)+\Pr(1998)\Pr(A|1998)+\Pr(1998)\Pr(A|1998)} \\
&= \frac{(0.16/0.54)\times 0.05}{(0.16/0.54)\times 0.05+(0.18/0.54)\times 0.02+(0.20/0.54)\times 0.03} \\
&= 0.4545
\end{aligned}$$

Note that the factor of 0.54 cancels from the numerator and the denominator, so we can also calculate the required probability as:

$$\Pr(1997|A) = \frac{0.16\times 0.05}{0.16\times 0.05+0.18\times 0.02+0.20\times 0.03} = 0.4545$$